that we have to deal with massive residual heat. The high-voltage operation in space also causes an insulation problem of the satellite surface under the ionospheric plasma environment.

One big problem would be the pointing and tracking of the debris. We have assumed a pointing accuracy of 0.1 μrad with a 1.6-m telescope. Accurately tracking a fast-moving object with such a large telescope is very challenging. In the numerical simulations in Sec. III, the steering speed required for the gimbaled telescope is calculated to be $\omega = 4.3$(deg/s) and $\dot{\omega} = 8.6$(deg/s^2) for debris at an altitude of 800 km and with 90 deg inclination. This laser satellite system has many elements in common with laser intersatellite communication, for which several space experiments are planned in the near future. With minor modifications to the experimental equipment, the debris pointing and tracking experiment can be conducted on the same platform. Because the proposed laser satellite is gigantic, this approach is a reasonable first step.

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References

[1]Griffin, M. D., andFrench, J. R., *Space Vehicle Design*, Education Series AIAA, Washington, DC, 1991, Chap. 3.

[2]Kantrovitz, A., "The Relevance of Space," *Astronautics and Aeronautics*, Vol. 9, No. 3, 1971, pp. 34–35.

[3]Bailly, M., and Perez, E., "The Pointing, Acquisition, and Tracking System of SILEX European Program: A Major Technological Step for Intersatellites Optical Communication," *SPIE Optical Engineering Reports*, Vol. 1417, 1991, pp. 142–157.

[4]"Report to the American Physical Society of the Study Group on Science and Technology of Directed Energy Weapons," *Reviews of Modern Physics*, Vol. 59, No. 3, 1987, pp. S1–S201.

Calculating the Number of Meteoroid Impacts onto a Gravity-Gradient-Stabilized Spacecraft

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Introduction

THE meteoroid shield assessment procedure[1] developed for Apollo implicitly assumes a randomly tumbling spacecraft stationary with respect to the meteoroid environment measured by Earth-based sensors, i.e., a spacecraft with no orbital speed. In order to determine the effect of this assumption on the reduction of spacecraft-borne meteoroid sensor data and on spacecraft meteoroid shield assessment, Kessler[2] performed an analysis showing that the spacecraft orbital speed results in negligible increase of the calculated flux onto a randomly tumbling spacecraft. Hence, most analysts have not included the effect of spacecraft speed in their analyses.

With the approaching retrieval of the gravity-gradient-stabilized spacecraft LDEF, Zook[3] reexamined the effect of spacecraft speed on the meteoroid flux onto a spacecraft. He showed that even though there is only a small increase in number of impacts onto the vehicle as a whole, the number of impacts onto the fore portions of the spacecraft is an order of magnitude larger than onto the aft. Thus the effect of spacecraft speed is crucial to understanding the distribution of impacts around a gravity-gradient-stabilized spacecraft such as LDEF and the Space Station.

Two assessment procedures for gravity-gradient-stabilized spacecraft have been published. Sullivan and McDonnell[4] specialized their procedure to the case of a meteoroid environment with a single speed. Zook,[3] using a procedure similar to that suggested by Kessler,[2] made a more realistic calculation by starting from the measured meteor closing-speed probability density function. First he calculated small increments of the mean number of impacts on a spacecraft with velocity v_s in the stationary spacecraft reference frame for small increments of meteoroid closing speed v and polar approach angle ψ. Second, he transformed the coordinate increments from the stationary spacecraft reference frame to coordinate increments in the orbiting spacecraft reference frame, in which the spacecraft is at rest. Last, he sorted the increment of the mean number of impacts into the appropriate orbiting spacecraft coordinate bins, where a running sum for the mean number of impacts was kept for each bin.

An alternative to Zook's numerical procedure is described here. Because the meteoroid environment may be described in terms of continuous probability density functions, it is possible to use a Jacobian to describe the coordinate transformation and the integral calculus to calculate the mean number of impacts. The results from this approach are compared with Zook's calculation of the ratio of the fluxes on the fore and aft facing sides of a flat plate.

Meteoroid State

The state of the meteoroid is defined as its closing speed and its approachangle with respect to the reference surface that registers the impacting meteoroid flux. Thus, the meteoroid state is a function of the shape of the reference surface and the velocity of the reference surface relative to the meteoroid environment. A complete description of the meteoroid state is only available for meteors. Kessler[2] has shown that the approach angles of meteoroids entering the Earth's atmosphere are isotropically distributed with respect to the Earth, and consequently the meteoroid closing speed is independent of approach angle. (Note that any point on the surface of the Earth may not see an isotropic distribution of approach angles, but the Earth's surface as a whole will.) Thus the probability of a meteoroid having a certain state with respect to the Earth is the product of the isotropic approach-angle probability density function with the closing-speed probability density function.

This closing-speed probability density function $g(v)$ was measured for photographic meteors by Hawkins and Southworth.[5] These data were later analyzed by Erickson[6] to eliminate observational selection effects, and the resulting graphical closing-speed probability density function was fitted by Zook[7] with the equations

$$g(v) = \begin{cases} 0.1120, & 11.1\ \text{km/s} \le v \le 16.3\ \text{km/s} \\ 3.328 \times 10^5 v^{-5.34}, & 16.3\ \text{km/s} \le v \le 55.0\ \text{km/s} \\ 1.695 \times 10^{-5}, & 55.0\ \text{km/s} \le v \le 72.2\ \text{km/s} \end{cases} \tag{1}$$

To make the connection with spacecraft, note that a randomly tumbling flat plate will also see an isotropic distribution of approach angles. Therefore, the function $g(v)$ can be thought of as the closing-speed probability density function for an imaginary randomly tumbling flat plate stationary with respect to the Earth, with v the meteoroid closing speed measured relative to the randomly tumbling flat plate.

Surprisingly, the meteoroid state description for meteoroids striking a fictitious stationary randomly tumbling flat plate can be used to make acceptable estimates of the flux onto an orbiting randomly tumbling flat plate. Kessler[2] has shown the ratio of the flux on an orbiting randomly tumbling flat plate to the flux on a stationary randomly tumbling flat plate is

$$1 + v_s^2 \int_{v_{\min}}^{v_{\max}} \frac{g(v)}{v^2}\, dv \tag{2}$$

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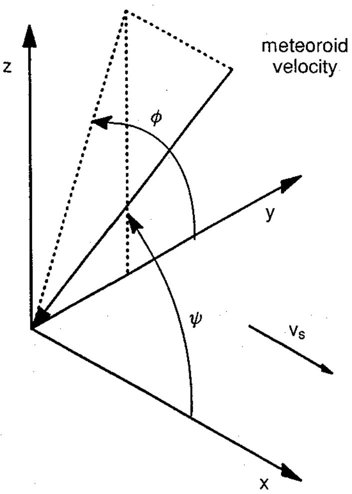

Fig. 1 Polar coordinate system for the stationary spacecraft reference frame.

For a randomly tumbling spacecraft in a circular orbit 400 km above the surface of the Earth integration of Eq. (2) gives 1.084—a small factor relative to the uncertainty in the meteoroid environment measurements. For future reference, Eq. (2) is called the spacecraft velocity flux focusing factor V_E.

The analysis of the distribution of impacts on a gravity-gradient-stabilized spacecraft also starts with the approach-angle probability density function. Unfortunately the necessary distribution of meteor radiants with respect to the synodic-year sky, for constant-mass meteors, is poorly known because of strong observational selection effects. (See Ref. 3.) In order to get around this difficulty, Zook[3] has hypothesized that the effect of spacecraft motion on the meteoroid state with respect to a gravity-gradient-stabilized spacecraft can be thought of as composed of two parts. The first part is the orientation of the spacecraft with respect to the synodic-year sky. During a sufficiently long period of time, the precession of the satellite's orbit and the tilt of the Earth's equatorial plane with respect to the ecliptic results in the spacecraft pointing in many directions with respect to the synodic-year sky. This imaginary reference frame in which the spacecraft is not moving, but has pointed in a large number of directions, was assumed by Zook[3] to correspond to the stationary spacecraft reference frame in which the meteoroid approach angles are isotropically distributed over a randomly tumbling spacecraft. This assumption is reasonable and is also made here. The second part of the spacecraft motion's effect on the meteoroid state is the speed of the spacecraft through the isotropic environment that would impact a spacecraft at rest in the stationary spacecraft reference frame. The frame of reference in which the spacecraft is at rest is referred to as the orbiting spacecraft reference frame, as it was for randomly tumbling spacecraft.

A set of three Cartesian axes are defined for the stationary spacecraft reference frame, as shown in Fig. 1. This coordinate system is at rest with respect to the stationary spacecraft reference frame; hence the spacecraft will have speed v_s in these coordinates. To simplify matters, these coordinate axes are chosen so that spacecraft motion is along the x axis in the positive direction. Due to the axial symmetry of the spacecraft-velocity-focused meteoroid environment, the meteoroid velocity vector is referenced to the polar coordinates shown in Fig. 1. The meteoroid velocity vector is defined to make polar angle ψ with the x axis. The projection of the meteoroid velocity vector onto the y-z plane is defined to make azimuthal angle ϕ with the y axis. In this polar coordinate system the meteoroid polar-approach-angle probability density function $h(\psi)$ is given by the well-known relation $\frac{1}{2}\sin\psi\,\mathrm{d}\psi$ for isotropic distributions in polar coordinates.

A set of three Cartesian coordinate axes are also defined for the orbiting spacecraft reference frame. Coordinates in the orbiting spacecraft reference frame will be primed to distinguish them from coordinates in the stationary spacecraft reference frame. The primed orbiting spacecraft coordinates travel with the spacecraft and are oriented so that the x' axis is parallel to the x axis, the y' axis is parallel to the y axis, and the z' axis is parallel to the z axis. The meteoroid velocity vector makes the polar angle ψ' with the x' axis, and the projection of the meteoroid velocity vector onto the y'-z' plane is defined to make azimuthal angle ϕ' with the y' axis.

Mean Number of Impacts onto an Orbiting Spacecraft

The calculation of the mean number of impacts onto an orbiting spacecraft starts with the number of meteoroids per unit volume, S, because S is independent of the reference frame. For a simple meteoroid environment with one speed and one approach angle, S times the meteoroid speed v' gives the cross-sectional area flux F'_{sp} of meteoroids with speed v' onto a sphere at rest with respect to the orbiting spacecraft reference frame. Generalizing the above observation to a distribution of speeds and approach angles and solving for the increment of S composed of meteoroids with speeds between v' and $v'+\mathrm{d}v'$ and polar approach angles between ψ' and $\psi'+\mathrm{d}\psi'$ onto a reference sphere at rest with respect to the orbiting spacecraft reference frame gives $(F'_{\mathrm{sp}}/v')p(v',\psi')\,\mathrm{d}v'\,\mathrm{d}\psi'$, where $p(v',\psi')$ is the meteoroid-state bivariate probability density function in orbiting spacecraft coordinates for impacts onto a reference sphere at rest with respect to the orbiting spacecraft reference frame.

If the differential range of meteoroid speed and approach angles in the orbiting spacecraft reference frame considered above istransformed to the stationary spacecraft reference frame, then by definition the increment of S will result in the meteoroid flux onto a sphere at rest in the stationary spacecraft reference frame, F_{sp}. That is, the increment of S is $(F_{\mathrm{sp}}/v)g(v)h(\psi)\,\mathrm{d}v\,\mathrm{d}\psi$. Setting the two results for the increment of S equal to each other gives,

$$\frac{F'_{\mathrm{sp}}}{v'}p(v',\psi')\,\mathrm{d}v'\,\mathrm{d}\psi' = \frac{F_{\mathrm{sp}}}{v}g(v)h(\psi)\,\mathrm{d}v\,\mathrm{d}\psi \tag{3}$$

Multiply both sides of Eq. (3) by v' and integrate over all possible meteoroid states. The left-hand side of Eq. (3) reduces to F'_{sp} because the meteoroid-state bivariate probability density function is normalized. Multiply F'_{sp} by the cross-sectional area of the sphere, A, and by the duration of exposure, Δt, to obtain the mean number of impacts onto the reference sphere at rest in the orbiting spacecraft reference frame (i.e., a sphere with orbital speed v_s), or

$$\int_0^{\pi}\int_{v_{\min}}^{v_{\max}} F_{\mathrm{sp}}\frac{v'}{v}A\,\Delta t\,g(v)h(\psi)\,\mathrm{d}v\,\mathrm{d}\psi \tag{4}$$

The coordinate transformations from the meteoroid state in the stationary spacecraft reference frame (ψ, v), to the meteoroid state in the orbiting spacecraft reference frame (ψ', v') will be denoted as $\psi = \Psi(\psi', v')$ and $v = V(\psi', v')$. Then the transformation of Eq. (4) to orbiting spacecraft coordinates is given by

$$\int_0^{\pi}\int_{V'[\psi',v_{\min}]}^{V'[\psi',v_{\max}]} F_{\mathrm{sp}}\frac{v'}{V[\psi',v']}A\,\Delta t\,g(V[\psi',v'])$$
$$\times h(\Psi[\psi',v'])|J|\,\mathrm{d}v'\,\mathrm{d}\psi' \tag{5}$$

where the limits of integration of the inner integral are given by the function $v' = V'[\psi', v]$ and the parameter $|J|$ is the absolute value of the Jacobian of the coordinate transformation,

$$J = \frac{\partial\Psi}{\partial\psi'}\frac{\partial V}{\partial v'} - \frac{\partial\Psi}{\partial v'}\frac{\partial V}{\partial\psi'} \tag{6}$$

from the stationary spacecraft reference frame to the orbiting spacecraft reference frame.

The derivations of the coordinate transformations $V(\psi', v')$, $V'(\psi', v)$, and $\Psi(\psi', v')$ start with the well-known Galilean coordinate transformation from mechanics and the coordinate system conventions defined in Fig. 1, listed below for the x and y components:

$$v_x = v'_x - v_s \tag{7}$$

$$v_y = v'_y \tag{8}$$

Expressing Eqs. (7) and (8) in the polar coordinates defined in Fig. 1 gives

$$v \cos \psi = v' \cos \psi' - v_s \tag{9}$$

$$v \sin \psi \cos \phi = v' \sin \psi' \cos \phi' \tag{10}$$

The azimuthal angles are unchanged by the coordinate transformation, so $\phi = \phi'$. Therefore Eq. (10) reduces to,

$$v \sin \psi = v' \sin \psi' \tag{11}$$

To solve for $V(\psi', v')$ square both Eqs. (9) and (11), add the results and take the square root to obtain

$$v = \left(v'^2 + v_s^2 - 2v'v_s \cos \psi'\right)^{\frac{1}{2}} \tag{12}$$

To solve for $V'(\psi', v)$, note that Eq. (12) squared is a quadratic equation for v' with the solution

$$v' = v_s \cos \psi' + \sqrt{v_s^2 \cos^2 \psi' - (v_s^2 - v^2)} \tag{13}$$

To solve for $\Psi(\psi', v')$, divide Eq. (9) by v and take the arccosine to get

$$\psi = \cos^{-1}\left(\frac{v' \cos \psi - v_s}{v}\right) \tag{14}$$

where the v in the denominator is given by Eq. (12).

Finally, the Jacobian is solved for by substituting Eqs. (14) and (12) into Eq. (6) to obtain

$$J = \frac{v'}{v} \tag{15}$$

With the above results, Eq. (5) can be further simplified. Replace the Jacobian with Eq. (15) and $h(\psi)$ with $(\sin \psi)/2$ and substitute Eq. (11) into the result to obtain

$$\int_0^\pi \int_{V'[\psi', v_{\min}]}^{V'[\psi', v_{\max}]} F_{\text{sp}}\left(\frac{v'}{V[\psi', v']}\right)^3 A\,\Delta t \times g(V[\psi', v'])\frac{\sin \psi'}{2}\,\mathrm{d}v'\,\mathrm{d}\psi' \tag{16}$$

Equation (16) normalized by $F_{\text{sp}} A\,\Delta t$ is the velocity focusing factor in orbiting spacecraft coordinates. That factor was numerically integrated and compared with Kessler's equation for the velocity focusing factor in stationary spacecraft coordinates, which is repeated as Eq. (2) in this paper. Agreement was obtained to within the tolerance set for the numerical integrations (six significant figures for the smallest tolerance calculated), confirming the accuracy of the coordinate transformations and the Jacobian derived here.

The mean number of impacts onto an oriented flat plate is obtained from Eq. (16) by multiplying the integrand of Eq. (16) by the area the flat plate presents to meteoroids. The presented area of the flat plate is equal to the product of the area A with the cosine of the angle β the meteoroid makes with respect to the plate normal. Therefore, the mean number of impacts onto an oriented flat plate is

$$\int_0^\pi \int_{V'[\psi', v_{\min}]}^{V'[\psi', v_{\max}]} F_{\text{sp}}\left(\frac{v'}{V[\psi', v']}\right)^3 A[\cos \beta]^+\,\Delta t \times g(V[\psi', v'])\frac{\sin \psi'}{2}\,\mathrm{d}v'\,\mathrm{d}\psi' \tag{17}$$

The notation $[\ ]^+$ denotes that only impacts on the upside of the plate are counted, i.e., whenever $\cos \beta \leq 0$, set the quantity in square brackets equal to zero.

Equation (17) can be used to calculate the ratio of the flux on a forward-facing flat plate to the flux on an aft-facing flat plate, by taking the ratio of Eq. (17) evaluated from 0 to $\pi/2$ (instead of from 0 to π) to Eq. (17) evaluated from $\pi/2$ to π. Zook[3] obtained a ratio 7.2 for a flat plate at 460-km altitude and $g(v)$ given by Eq. (1), whereas a ratio 9.04 was obtained with Eq. (17), when Earth shadowing was taken into account.

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References

[1]Frost, V. C., "Meteoroid Damage Assessment," NASA SP-8042, May 1970.

[2]Kessler, D. J., "A Guide to Using Meteoroid-Environment Models for Experiment and Spacecraft Design Applications," NASA TN D-6596, March 1972.

[3]Zook, H. A., "Deriving the Velocity Distribution of Meteoroids from the Measured Meteoroid Impact Directionality on the Various LDEF Surfaces," *LDEF—69 Months in Space, First Post-Retrieval Symposium*, NASA CP-3134, Part 1, 1991, pp. 569–580.

[4]Sullivan, K., and McDonnell, J. A. M., "LDEF Flux Anisotropy: Dynamic Modelling and Flux Transformations to Define the Interplanetary Micrometeoroid Environment," *Hypervelocity Impacts in Space*, University of Kent Press, Canterbury, England UK, 1993, pp. 126–132.

[5]Hawkins, G. S., and Southworth, R. B., "Statistics of Meteors in the Earth's Atmosphere," *Smithsonian Contributions to Astrophysics*, Vol. 2, No. 11, 1958, pp. 349–364.

[6]Erickson, J. E., "Velocity Distribution of Sporadic Photographic Meteors," *Journal of Geophysical Research*, Vol. 73, No. 12, 1968, pp. 3721–3726.

[7]Zook, H. A., "The State of Meteoritic Material on the Moon," *Proceedings of the Lunar Science Conference, 6th*, 1975, pp. 1653–1672.

Analytical Solution for Controls, Heats, and States of Flight Trajectories

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Nomenclature

C_D = drag coefficient, where $C_{D_t} = C_{D_a} + C_{D_r}$
C_L = lift coefficient, where $C_{L_t} = C_{L_a} + C_{D_r}$
C_Q = heat constant = $1.83 \times 10^{-8} R_n^{-1/2}(1 - h'_W/h'_0)$, $(\text{J/cm}^2)/(\text{kg/m}^3)^{\frac{1}{2}}(m^3/s^3)$, for stagnation point, where R_n is in meters, h'_W = enthalpy at wall, and h'_0 = total enthalpy
D = drag, where $D_t = D_a + D_r$, N
E = aerodynamic-lift-to-aerodynamic-drag ratio, $L_3/D_a = C_{L_a}/C_{D_a}$
E^* = maximum E, $(L_a/D_a)_{\max} = C^*_{L_a}/C^*_{D_a}$
f_a = function of the aerodynamic control, $C_{D_a}/C^*_{L_a} = 1 + \lambda_a^2/2E^*$
f_t = function of the total aerodynamic control, where $f_t = f_a + f_r = 1 + \lambda_t^2/2E^*$
g = acceleration due to gravity, m/s^2
h = geometric altitude from the earth's equator, km
I_{sp} = specific impulse, s
L = lift, where $L_t = L_a + L_r$, N
M = Mach number
m = mass of the vehicle at any time, kg
Q = heat load per unit area, kJ/cm^2
r = position of the vehicle's mass center with respect to the earth's center, m or km
S = vehicle aerodynamic reference area, m^2
T_t = total (or gross) thrust, N
T_n = net thrust, $T_t - D_r$, N
t = time, s

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